# Accurate Experimental Characterization of Three-Ports

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## ABSTRACT

An accurate procedure is reported for experimentally characterizing microwave devices using two-port measurements. Reflection measurements only are used to determine three-port reflection parameters and primarily transmission measurements to determine three-port transmission parameters, thus considerably reducing the sensitivity of the procedure. No assumptions about the three-port device is made for the procedure. The results are compared to Woods' renormalization method.

## INTRODUCTION

Measurements of three-port network parameters have been used in characterizing transistors [1], [2], [3], and passive ructures [4], and in calibrating power sensors [5]. Development of three-port parameters is not straightforward as they must be constructed from a number of two-port measurements.

Several algorithms have been presented for assembling the three-port parameters from a number of two-port measurements. The simplest approach is to terminate the unused port in the reference impedance of the measurement system during a two-port scattering parameter measurement [1]. In this approach, the effect of impedance mismatches at the unused port are ignored. This can introduce significant errors particularly when fixturing cannot be ignored.

### Woods

Woods considered this problem in a number of papers culminating in a treatise on the development of N-port parameters from two port measurements [6]. His was a method of multiple renormalizations taking into the account the actual impedance presented to the unused port(s) during a two-port measurement. The technique is non-iterative and behaves reasonably well when the impedance presented to the ports are close to the system reference impedance. As will be seen, problems arise when the impedance at the unused port varies significantly from the two-port measurement reference impedance.

### Speciale

Speciale applied TSD (renamed as Super TSD) techniques to N-port measurements in 1977 and included a leakage term in the N-port error model [7]. The new model for the error network treats one path from an input to a single output as a direct path and all others paths from other input ports to the same output as leakage. This technique works well for network analyzer calibration when leakages do occur from the error network, but it does not directly account for mismatches at the non-tested ports. There are also limits due to the TSD standards in that a reflectionless line is required as a standard.

### Sharma and Gupta

In 1981, Sharma and Gupta used desegmentation techniques to deembed multiports [4]. Instead of a large error network connecting all ports of the multiport, they assumed a number of independent two-port error networks at each port of the DUT. The individual two-port error networks must be characterized and the parameters of the embedded N-port parameters determined. However, their method is not concerned with the construction of three-port parameters from measurement.

### Gruner

Gruner introduced resonant and nonresonant methods to measure S parameters of multiports [10]. The measurement technique involves varying line lengths, which is not always feasible, as well as assuming that the discontinuity of the input lines are small. This, of course, negates application to microstrip circuits where coaxial to microstrip transitions are significant. Furthermore, the phase of the transmission parameters are indeterminate by $\pm\pi$. His resonant method, as well as a similar method presented by Easter et al. [11], applies to measurement at just a few discrete frequencies.

Only the renormalization method of Woods is directly applicable to the construction of three-port network parameters from two-port measurements. Woods uses a minimum number of measurements — two-port measurements between each pair of terminals. Errors accumulate as multiple combinations of two-port parameters are required to determine each of the three-port scattering parameters. The accumulated errors are significant when the magnitudes of three-port parameters vary greatly or when the unused port terminations are not close to the measurement reference impedance (usually 50 $\Omega$).

The purpose of this paper is to present an experimental procedure suited to a more accurate construction of three-port scattering parameters from two-port measurements. The impedance presented to the unused port during a two-port measurement need not be close to the reference impedance of the measurement system. In our approach two sets of terminations are used to obtain near optimum deembedding whereby reflection measurements solely are used to determine reflection parameters and primarily transmission measurements are used to determine transmission parameters. Hence the technique is called the multiple termination method (MTM) for characterizing three-port networks. Results for a microstrip tee are compared to results obtained using Woods' renormalization technique.

## CONSTRUCTION OF THREE-PORT PARAMETERS

Three-port calibration and characterization of a device must be performed using two-port measurements with the third port terminated. The relationship between the S parameters of the three port, $S_{ij}$, the measured two-port S

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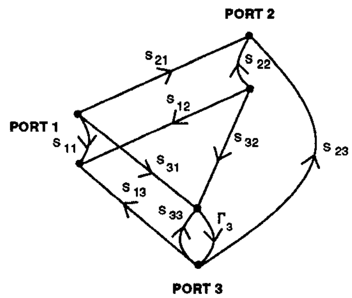


Figure 1:
Signal flow graph of a three port device with port 3 terminated by reflection coefficient, Γ.

parameters, $S_{ij}^M$, and the reflections of the terminations $\Gamma_k$ is developed in the following using the signal flowgraph of the partially terminated three port, see Fig. 1.

Using Mason's non-touching loop theorem, [12], the two-port reflection S parameter at the $i^{th}$ node is

$$\begin{aligned} {}^{mk}S_{ii}^M &= S_{ii} + \frac{S_{ki}\,{}^m\Gamma_k\, S_{ik}}{1 - S_{kk}\,{}^m\Gamma_k} \\ &= S_{ii}\,(1 - S_{kk}\,{}^m\Gamma_k) + S_{ik}\,{}^m\Gamma_k S_{ki} + {}^{mk}S_{ii}^M\, S_{kk}\,{}^m\Gamma_k \end{aligned} \tag{1}$$

The measured two-port transmission S parameter between nodes $i$ and $j$ can be found in the same way:

$$\begin{aligned} {}^{mk}S_{ij}^M &= S_{ij} + \frac{S_{kj}\,{}^m\Gamma_k\, S_{ik}}{1 - S_{kk}\,{}^m\Gamma_k} \\ &= S_{ij}\,(1 - S_{kk}\,{}^m\Gamma_k) + S_{kj}\, S_{ik}\,{}^m\Gamma_k + {}^{mk}S_{ij}^M\,{}^m\Gamma_k S_{kk} \end{aligned} \tag{2}$$

In both equations, $M$ indicates that the S parameter is a measured quantity, and $m$ and $k$ are indices distinguishing different terminations ${}^m\Gamma_k$ at port $k$. The equations (1) and (2) describe 3 sets of two-port S parameter measurements with $i, j, k = 1, 2, 3$; $i \neq j \neq k$. This leads to 12 coupled nonlinear equations which have multiple solutions. The equations can also be poorly conditioned as the measured two-port S parameters may only be determined to 1% accuracy. This is especially so if the magnitude of one three-port S parameter is much less than that of another, since then the small S parameters may be lost in the equation solution process. That is, it may not be possible to determine S parameters that are small relative to other S parameters.

In general, good accuracy is obtained when $S_{ii}$ ($S_{ij}$) is evaluated primarily in terms of ${}^{mk}S_{ii}^M$ ($S_{ij}^M$) measurements as, usually, they are of the same order. This requires that two different reflection standards be used at each port (so that $m = 1,2$) and that two-port S parameter measurements be taken for all two port combinations (that is $i, j, k = 1, 2, 3$, $i \neq j \neq k$). This leads to 24 equations which can be solved as a linear set of equations.

Multiplying the equation for ${}^{1k}S_{ii}^M$ ((1) with $m = 1$) by ${}^2\Gamma_k$ and subtracting this from the equation for ${}^{2k}S_{ii}^M$ ((1) with $m = 2$) multiplied by ${}^1\Gamma_k$ gives

$$\begin{aligned} &\left({}^1\Gamma_k\,{}^{2k}S_{ii}^M - {}^2\Gamma_k\,{}^{1k}S_{ii}^M\right) = \\ &\qquad S_{ii}\left({}^1\Gamma_k - {}^2\Gamma_k\right) + S_{kk}\,{}^1\Gamma_k\,{}^2\Gamma_k\left({}^{2k}S_{ii}^M - {}^{1k}S_{ii}^M\right) \end{aligned} \tag{3}$$

Similarly

$$\begin{aligned} &\left({}^1\Gamma_i\,{}^{2i}S_{kk}^M - {}^2\Gamma_i\,{}^{1i}S_{kk}^M\right) = \\ &\qquad S_{kk}\left({}^1\Gamma_i - {}^2\Gamma_i\right) + S_{ii}\,{}^1\Gamma_i\,{}^2\Gamma_i\left({}^{2i}S_{kk}^M - {}^{1i}S_{kk}^M\right) \end{aligned} \tag{4}$$

Subtracting (3) times $({}^1\Gamma_i - {}^2\Gamma_i)$ from (4) times ${}^1\Gamma_k^2\Gamma_k$ $\left({}^{2k}S_{ii}^M - {}^{1k}S_{ii}^M\right)$ and rearranging yields

$$S_{ii} = \frac{{}^1\Gamma_k\,{}^2\Gamma_k\left({}^{2k}S_{ii}^M - {}^{1k}S_{ii}^M\right)\left({}^1\Gamma_i\,{}^{2i}S_{kk}^M - {}^2\Gamma_i\,{}^{1i}S_{kk}^M\right) - \left({}^1\Gamma_i - {}^2\Gamma_i\right)\left({}^1\Gamma_k\,{}^{2k}S_{ii}^M - {}^2\Gamma_k\,{}^{1k}S_{ii}^M\right)}{{}^1\Gamma_k\,{}^2\Gamma_k\left({}^{2k}S_{ii}^M - {}^{1k}S_{ii}^M\right){}^1\Gamma_i\,{}^2\Gamma_i\left({}^{2i}S_{kk}^M - {}^{1i}S_{kk}^M\right) - ({}^1\Gamma_i - {}^2\Gamma_i)({}^1\Gamma_k - {}^2\Gamma_k)} \tag{5}$$

which as expected reduces to

$$S_{ii} = {}^{2k}S_{ii}^M \tag{6}$$

when the second termination at port $k$ is matched, i.e. ${}^2\Gamma_k = 0$. Equation (5) describes two $S_{ii}$ solutions for each $i$, as $j$ can be substituted for $k$.

The transmission parameters can be obtained following a similar approach. Combining the two equations obtained from (2) with $m = 1$ and 2 and rearranging

$$S_{ij} = \frac{\left({}^1\Gamma_k\,{}^{2k}S_{ij}^M - {}^2\Gamma_k\,{}^{1k}S_{ij}^M\right) - S_{kk}\,{}^1\Gamma_k\,{}^2\Gamma_k\left({}^{2k}S_{ij}^M - {}^{1k}S_{ij}^M\right)}{{}^1\Gamma_k - {}^2\Gamma_k} \tag{7}$$

where $S_{kk}$ is found from (5). Again, if ${}^2\Gamma_k = 0$, the S parameter is determined from a single measurement.

$$S_{ij} = {}^{2k}S_{ij}^M \tag{8}$$

The S parameters of a three-port are found by multiple application of (5) and (7) for all combinations of $i$, $j$ and $k$ such that $i, j, k = 1, 2, 3$ and $i \neq j \neq k$.

Both the multiple termination method (MTM) presented here and Woods' renormalization technique were verified using data synthesized from a commercial linear circuit CAD program. The differences between the two methods can only be due to different sensitivities to finite experimental precision.

## Experimental Characterization

### Deembeding

It has been shown that coaxial to microstrip transitions are reasonably first order symmetric structures [14]. Fig. 2 shows such a transition. It is obvious from the geometry that large discontinuities exist between the coaxial reference plane and the microstrip reference plane causing a fixturing effect. This fixturing can be removed using enhanced through symmetric line (ETSL) deembedding [14].

Fig. 3 is a simple model of an embedded three-port microstrip device. We see fixturing can have a large effect during the measurement process. Therefore, deembeding must take place. This is a simple procedure since all the measurements are of a two-port type and the symmetry of the coaxial to microstrip transition allows us to deembed without many extra measurements of standards.

### Terminations

In the derivation for the characterization of a three-port in the last section, the reflection coefficient of the termination on the unused port, as seen by the deembeded microstrip reference plane, is necessary. However, the termination has an intervening transition before the microstrip reference plane. This can be seen in fig. 4.

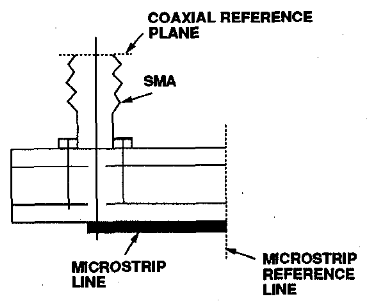


Figure 2:
Coax-to-microstrip transition showing reference planes.

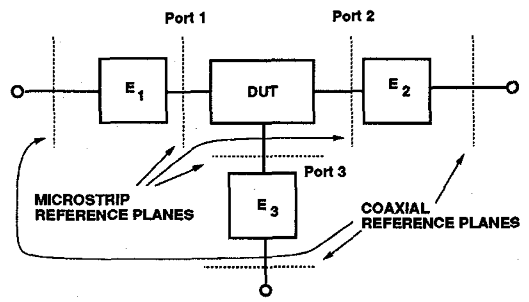


Figure 3:
Simple model of a three port device.

This circuit can be used to partially deembed the termination for the data needed in the three-port algorithm. The result for a 50Ω load is shown in fig. 5.

Three-port

A printed circuit board (PCB) microstrip tee in the configuration in fig. 6 was measured two ports at a time with the unused port terminated in either 50Ω or a short. Fig. 7 is a sample of this data. This data was deembedded with the ETSL technique as described in [14].

With the deembedded data and the partially decascaded termination data that the embedded circuit sees on the extraneous port, we now have all the data required to fully characterize the microstrip tee. A total of 14 measurements are required, recognizing the reciprocity of the circuit in this case.

Fig. 8 is a portion of the results that characterize the microstrip tee in the configuration of fig. 6. It is interesting

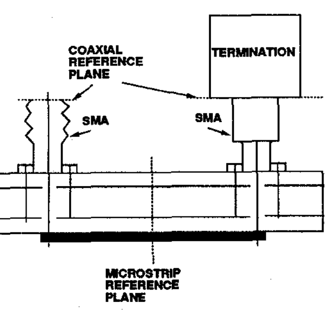


Figure 4:
Circuit to measure a termination for partial deembedment.

to note that the reflection parameters are around a value of approximately 0.33 which coincides with a 50Ω transmission line looking into parallel 50Ω loads. Also note that $S_{11}$ and $S_{22}$ track together closely, therefore $S_{22}$ was omitted from the plot, while $S_{33}$ loosely follows. This mimicry decreases with frequency, probably due to the increasing coupling of the base arm with the two cross arms. The reciprocity that is expected due to the physical configuration of the circuit is evident, although no assumption of such was made in the algorithm. The spikes that appear are probably due to resonances from the transmission line lengths of the arms in the tee as well as interactions of the inductive and capacitive coupling in the arms. These inductive and capacitive differences are more evident at the high frequencies.

Comparison with Renormalization Method

The raw data from the microstrip tee was also processed using algorithms developed by Wood [6]. He utilized multiple renormalization schemes that evolved into extensive matrix manipulations. Fig. 9 are the results with a 50Ω termination at the unused port. The technique was also used with a nominal short for a termination, but the resulting deembedded s parameters were erratic. This was due to large uncertainty errors of the renormalized S parameters when the impedance has a large reactive component.

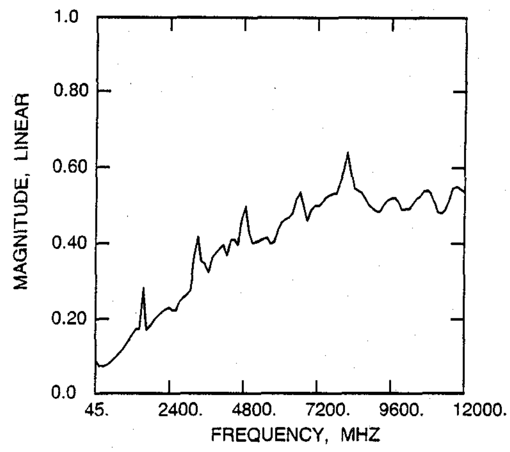


Figure 5:
Reflection coefficient of a nominal 50 Ω load including the coax-microstrip transition.

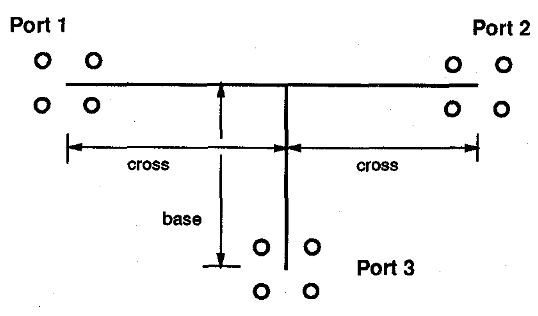


Figure 6:
Configuration of a microstrip tee with arms on the same plane of a PCB.

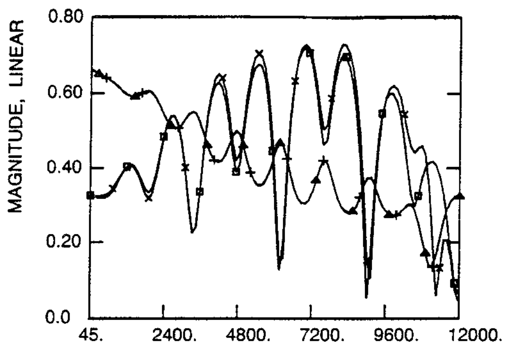


Figure 7:

Embedded S parameters of a microstrip tee with port 3 terminated in nominal 50 Ω load. Magnitude of (□) $S_{11}$, (△) $S_{12}$, (+) $S_{21}$, and (×) $S_{22}$.

The reflection results from the renormalization with the 50 Ω termination are generally similar to the MTM outcome up to 4.8 Ghz. Past that point, the data diverges greatly. A possible cause for this variance is Wood's dependence on the measurement of the termination. In fig. 5, we saw that the termination does not remain constant with frequency. Therefore, the reference impedance in Woods algorithm is frequency dependent. MTM uses two terminations and calculates to a universal 50 Ω reference and therefore is not as dependent upon any single measurement of a termination.

## Conclusion

An algorithm that accurately characterizes any three-port device was introduced. It is closed form and makes no assumptions (i.e. reciprocal transmission characteristics) upon the device under test, except for linearity. It takes into consideration the influences of reflections from untested ports and returns a full characterization of the device as it is seen in the circuit. MTM can easily be expanded to four or N-port devices with the necessary increase in different terminations. Symmetric deembedding can be easily used on the measured two-port data and any device can be fully characterized.

A comparison with Woods' renormalization method was done on a microstrip tee and an improvement upon accuracy was confirmed. There should be no reason that comparable comparisons with higher port devices should not have the same outcome.

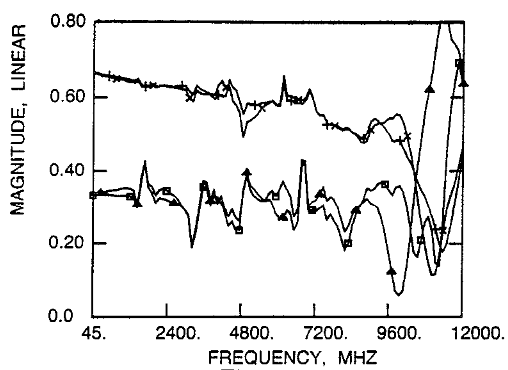


Figure 8:

S parameters of a microstrip tee using the MTM technique reported here. Magnitude of (□) $S_{11}$, (△) $S_{33}$,(+) $S_{31}$, and (×) $S_{32}$.

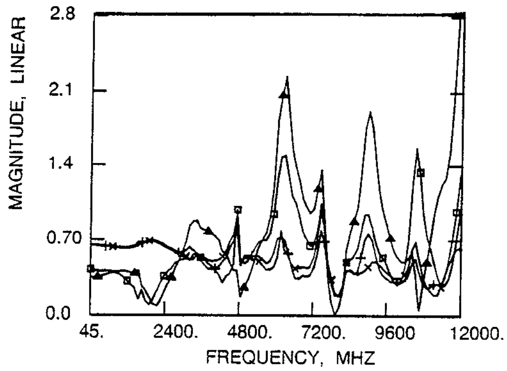


Figure 9:

S parameters of a microstrip tee characterized by using Woods' renormalization technique with nominal 50 ohm load on unused port. Magnitude of (□) $S_{31}$, (△) $S_{13}$, (+) $S_{32}$, and (×) $S_{23}$.

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